Technical Report

Distributed Nonlinear Control of Mobile Autonomous Multi-Agents

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Abstract

This paper studies the distributed nonlinear control of mobile autonomous agents with variable and directed topology. A new distributed nonlinear design scheme is presented for multi-agent systems modelled by double-integrators. With the new design, the outputs of the controlled agents asymptotically converge to each other, as long as a mild connectivity condition is satisfied. Moreover, the velocity (derivative of the output) of each agent can be restricted to be within any specified neighborhood of the origin, which is of practical interest for systems under such physical constraint. The new design is still valid if one of the agents is a leader and the control objective is to achieve leader-following. As an illustration of the generality and effectiveness of the presented methodology, the formation control of a group of unicycle mobile robots with nonholonomic constraints is revisited. Instead of assuming the point-robot model, the unicycle model is transformed into two double-integrators by dynamic feedback linearization, and the proposed distributed nonlinear design method is used to overcome the singularity problem caused by the nonholonomic constraint by properly restricting the velocities.

1 Introduction

Distributed control of multi-agent systems under communication constraints has attracted tremendous attention from the control community over the past ten years; see, for example, [42] using an adaptive gradient climbing strategy, [21, 12, 6, 4, 45] based on linear algebra and graph theory, [2, 52] using passivity and dissipativity theory, [15, 39, 19, 33, 50, 49, 40, 5] with Lyapunov methods, [16] using the nonlinear small-gain theorem, and [57, 58, 53] based on output regulation. The recent hot topics such as formation control, consensus, flocking, swarm, rendezevous and synchronization are all closely related to distributed coordinated control.

The distributed control problem for agents with second-order dynamics has been mainly studied from the perspective of second-order consensus and flocking. Considerable efforts have been devoted to solving the problems under switching information exchange topologies. Related results include [12, 43, 55, 44, 5]. Specifically, [12, 43] used potential functions to define Lyapunov functions and the topologies are allowed to be swiching but undirected. [44] presented a consensus result for double-integrator systems based on a refined graph theoretical method. [5] proposed a variable structure approach based consensus design method for systems with switching but always connected information exchange topology. Several recent results on distributed control can also be found in [60, 37, 31, 14, 47]. It should be pointed out that most of the papers mentioned above do not consider systems under physical constraints (e.g., saturation of velocities), for which specific distributed nonlinear designs are expected.

As a practical application of distributed control, the formation control of autonomous mobile agents aims forcing the agents to converge towards, and to maintain, specific relative positions, by using available information, e.g., relative position measurements. Recent formation control results can be found in [8, 56, 41, 10, 11, 30, 1, 20, 28], to name a few. The earlier results, e.g., [8, 56], assume a tree sensing structure to avoid the technical difficulties caused by the loop interconnections. In [41, 10, 11, 30], the assumption of tree sensing structures is relaxed at the price of using global position measurements. An exception is the wiggling controller developed by [32] to drive the robots to stationary points, which does not use global position measurements of the robots. In the results of coordinated path following as presented in [1, 20, 28], the global position measurement issue can be easily addressed as each robot has access to its desired path. In our recent paper [35], thanks to the use of nonlinear small-gain techniques [23, 34], the requirement on global position measurements have been removed for formation control of unicycles with fixed sensing topologies.

This paper takes a step forward toward solving the distributed nonlinear control problem for multi-agent systems modelled by double-integrators under switching information exchange topology and velocity constraints. Our goal is to develop a new class of distributed nonlinear control laws to solve a strong output agreement problem, that is, the positions of the agents converge to each other and the velocities of the agents converge to the origin. From a practical point of view, we assume that the double-integrators interact with each other through output interconnections (more specifically, the differences of the outputs), for coordination. In this paper, an invariant set method is developed such that the strong output agreement problem is solvable if the information exchange digraph satisfies a mild connectivity condition. Moreover, the proposed nonlinear design can also effectively handle physical constraints such as velocity limitation, for which linear designs are usually not directly applicable without significant modifications.

Based on the new nonlinear distributed control design, in this paper, we also develop a new class of formation controllers for unicycle robots under switching sensing topology. Moreover, the new formation controller does not use global position measurements. To this end, we first transform the formation control problem into an output (position) agreement problem of double-integrator agents with dynamic feedback linearization technique; see, e.g., [7]. Then, we develop a new class of distributed control laws. To avoid the singularity problem caused by the nonholonomic constraint, we employ saturation functions in the distributed control design to restrict the linear velocities of the robots to be larger than zero. With the new design, the linear velocities of the robots can also be restricted to be less than certain desired values, which is of practical interest.

The rest of the paper is organized as follows. In Section 2, we give the problem formulation. Section 3 presents our main result on distributed nonlinear control design for output agreement. In Section 4, we develop a new class of distributed formation controllers for a group of unicycle mobile robots. In Section 5, we offer some concluding remarks. The appendix contains a technical lemma on convergence properties of a class of nonlinear systems and the proof of an important proposition for the distributed nonlinear control design.

2 Problem Formulation

In this paper, we study the problem of designing nonlinear distributed controllers for output agreement of a group of N double-integrator agents with switching topologies by distributed control:

$$\dot{\eta}_i = \zeta_i \quad (1)$$

$$\dot{\zeta}_i = \mu_i \quad (2)$$

where $[\eta_i, \zeta_i]^T \in \mathbb{R}^2$ is the state and $\mu_i \in \mathbb{R}$ is the control input. Usually, η_i and ζ_i represent the position and the velocity of agent i, respectively.

Strong Output Agreement Problem: Our goal is to design a class of distributed nonlinear control laws in the form of

$$\mu_i = \overline{\varphi}_i(\zeta_i, \xi_i) \quad (3)$$

$$\xi_i = \overline{\phi}_i^{\sigma(t)}(\eta_1, \ldots, \eta_N) \quad (4)$$

where $\sigma : [0, +\infty) \to \mathcal{P}$ is a piecewise constant signal representing switching information exchange topology with $\mathcal{P} \subset \mathbb{N}$ being a finite set representing all the possible information exchange topologies, $\overline{\varphi}_i : \mathbb{R}^2 \to \mathbb{R}$ and $\overline{\phi}_i^p : \mathbb{R}^N \to \mathbb{R}$ for each $p \in \mathcal{P}$, such that the following properties hold:

$$\lim_{t\to\infty} (\eta_i(t) - \eta_j(t)) = 0, \quad \text{for any } i, j = 1, \ldots, N, \quad (5)$$

$$\lim_{t\to\infty} \zeta_i(t) = 0, \quad \text{for } i = 1, \ldots, N. \quad (6)$$

Notice that the strong output agreement as defined above is stronger than the standard state agreement.

As a practical engineering application, we study the distributed formation control of a group of $N+1$ nonholonomic mobile robots under switching position measurement topology. For $i = 0, \ldots, N$, each i-th robot is modeled by the unicycle model:

$$\dot{x}_i = v_i \cos\theta_i \tag{7}$$

$$\dot{y}_i = v_i \sin\theta_i \tag{8}$$

$$\dot{\theta}_i = \omega_i \tag{9}$$

where $[x_i, y_i]^T \in \mathbb{R}^2$ represent the Cartesian coordinates of the center of mass of the i-th robot, $v_i \in \mathbb{R}$ is the linear velocity, $\theta_i \in \mathbb{R}$ is the heading angle, and $\omega_i \in \mathbb{R}$ is the angular velocity.

The robot with index 0 is the leader robot, and the robots with indices $1, \ldots, N$ are follower robots. We consider v_i and ω_i as the control inputs of the i-th robot for $i = 1, \ldots, N$. For system (7)–(9), the position of the leader robot is assumed to be accessible to (some of) the follower robots, and the control objective is to achieve the following properties:

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = d_{xij} \tag{10}$$

$$\lim_{t\to\infty} (y_i(t) - y_j(t)) = d_{yij} \tag{11}$$

with d_{xij}, d_{yij} being appropriate constants representing some desired relative positions, and

$$\lim_{t\to\infty} ((\theta_i(t) - \theta_j(t)) \bmod 2\pi) = 0 \tag{12}$$

with mod representing the modulo operation, for any $i, j = 0, \ldots, N$. For convenience of notations, let $d_{xii} = d_{yii} = 0$ for any $i = 0, \ldots, N$. We assume that $d_{xij} = d_{xik} - d_{xkj}$ and $d_{yij} = d_{yik} - d_{ykj}$ for any $i, j, k = 0, \ldots, N$. The formation control problem with $N = 1$ has been studied extensively in the past literature, see [22, 9] and the references therein. It should also be mentioned that, in a companion paper [35], we considered the fixed topology case and proposed a small-gain approach for formation control of unicycle robots in the presence of position measurement errors.

In this paper, we study the case where the relative position sensing topology may be switching in practical formation control systems. By means of the output agreement result developed for double-integrators, our goal here is to develop a new class of distributed coordinated controllers, which are capable of overcoming the problems caused by nonholonomic constraints and achieving the formation control objectives by using local relative position measurements under switching position sensing topologies as well as the velocity information of the leader.

3 Distributed Nonlinear Control for Output Agreement

In this section, we present a new class of distributed nonlinear controllers for strong output agreement of double-integrators under switching topologies.

We consider the distributed control system as a group of controlled agents interacting with each other. By designing the behavior of each individual agent under the influence of other agents, we show that the desired group behavior, strong output-agreement, is achievable. Specifically, in Subsection 3.1, we propose a design tool for the double-integrators, and analyze the influence of a specific class of external disturbances to them. When this tool is applied to distributed control design in Subsection 3.2, the external disturbance of each controlled agent is caused by its interaction with other agents. The strong output agreement is realizable with appropriately designed control laws.

3.1 Properties of a Class of Nonlinear Systems

Our strong output agreement result will be developed based on several properties of the following second-order nonlinear system:

$$\dot{\eta} = \zeta \tag{13}$$

$$\dot{\zeta} = \varphi(\zeta - \phi(\eta - \upsilon)) \tag{14}$$

where $[\eta, \zeta]^T \in \mathbb{R}^2$ is the state, $\upsilon \in \mathbb{R}$ is an external disturbance input, and $\varphi, \phi : \mathbb{R} \to \mathbb{R}$ are nonincreasing and locally Lipschitz functions.

System (13)–(14) can be considered as a double-integrator $\dot{\eta} = \zeta$, $\dot{\zeta} = \mu$ controlled by a feedback law $\mu = \varphi(\zeta - \phi(\eta - \upsilon))$. If η is available to feedback, then one may employ a backstepping control law $\mu = \varphi(\zeta - \phi(\eta))$ to stabilize the system [27]. Here, $\phi(\eta)$ is usually known as the virtual control law for the η-subsystem. For system (13)–(14), the external input υ is injected as an additive disturbance of the measurement of η. When this design is applied to distributed control, the υ is used to represent the interaction between the agents. Intuitively, with the design tool proposed in Proposition 1, we can guarantee that, given $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}]$, the output η ultimately converges to the region $[\underline{\upsilon}, \overline{\upsilon}]$. This means the agents do not enlarge the influence of others. Based on this, we are able to study the group behavior of the agents when they interact with each other in Subsection 3.2.

To make the paper self-contained, we recall that a function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be positive definite if $\gamma(s) > 0$ for all $s > 0$ and $\gamma(0) = 0$. $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{K}$ function, denoted by $\gamma \in \mathcal{K}$, if it is continuous, strictly increasing and $\gamma(0) = 0$; it is a $\mathcal{K}_\infty$ function, denoted by $\gamma \in \mathcal{K}_\infty$, if $\gamma \in \mathcal{K}$ and it satisfies $\gamma(s) \to \infty$ as $t \to \infty$. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to be a $\mathcal{KL}$ function, denoted by $\beta \in \mathcal{KL}$, if, for each $t \geq 0$, $\beta(\cdot, t)$ is a $\mathcal{K}$ function, and, for each $s \geq 0$, $\beta(s, \cdot)$ is non-increasing and $\beta(s, t) \to 0$ as $s \to \infty$. We use Id to represent the identity function. For a locally Lipschitz function f defined on $\mathbb{R}$, we use $\partial f(r)$ to represent the set of left and right derivatives of the function at r.

For convenience of notations, we define two new classes of functions. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is called an $\mathcal{I}^+\mathcal{L}$ function, denoted by $\beta \in \mathcal{I}^+\mathcal{L}$, if $\beta \in \mathcal{KL}$, $\beta(s, 0) = s$ for $s \in \mathbb{R}_+$, and for any specified $T > 0$, there exist continuous, positive definite and non-decreasing $\alpha_1, \alpha_2 < \text{Id}$, which depend on T, such that for all $s \in \mathbb{R}_+$, $\beta(s, t) \geq \alpha_1(s)$ for $t \in [0, T]$ and $\beta(s, t) \leq \alpha_2(s)$

for $t \in [T, \infty)$. A function $\beta : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is called an $\mathcal{IL}$ function, denoted by $\beta \in \mathcal{IL}$, if there exist $\beta', \beta'' \in \mathcal{I}^+\mathcal{L}$ such that for $t \geq 0$, $\beta(r,t) = \beta'(r,t)$ for $r \geq 0$, and $\beta(r,t) = -\beta''(-r,t)$ for $r < 0$. An intuitive example of $\mathcal{I}^+\mathcal{L}$ functions is $\beta(s,t) = se^{-at}$ defined for $s, t \in \mathbb{R}_+$ with a being a positive constant, while $\bar{\beta}(r,t) = re^{-at}$ defined for $r \in \mathbb{R}$ and $t \in \mathbb{R}_+$ is an $\mathcal{IL}$ function. It should be noted that any $\mathcal{I}^+\mathcal{L}$ is a $\mathcal{KL}$ function. The new notations are necessary when we want to avoid the finite-time convergence in a network with time-variable topology, which may lead to oscillation. The similar problem occurs in the state agreement problem of coupled nonlinear systems. See [33, Lemma 5.2] for some details.

Proposition 1 *If $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}]$ with $\underline{\upsilon} \leq \overline{\upsilon}$ being constants, and if functions φ and ϕ satisfy*

$$\varphi(0) = \phi(0) = 0, \tag{15}$$

$$\varphi(r)r < 0, \ \ \phi(r)r < 0 \ \text{for } r \neq 0, \tag{16}$$

$$\sup_{r \in \mathbb{R}} \{\max \partial\varphi(r)\} < 4 \inf_{r \in \mathbb{R}} \{\min \partial\phi(r)\}, \tag{17}$$

then system (13)–(14) *has the following properties:*

1. *There exist strictly decreasing and locally Lipschitz functions $\underline{\psi}, \overline{\psi} : \mathbb{R} \to \mathbb{R}$ satisfying $\underline{\psi}(0) = \overline{\psi}(0) = 0$ such that*

$$S(\underline{\upsilon}, \overline{\upsilon}) = \left\{(\eta, \zeta) : \underline{\psi}(\eta - \underline{\upsilon}) \leq \zeta \leq \overline{\psi}(\eta - \overline{\upsilon})\right\}$$

   is an invariant set of system (13)–(14)*;*

2. *For any specified initial state $(\eta(0), \zeta(0))$, there exist a finite time t_1 and constants $\underline{\mu}, \overline{\mu} \in \mathbb{R}$ such that*

$$\underline{\psi}(\eta(t_1) - \underline{\mu}) \leq \zeta(t_1) \leq \overline{\psi}(\eta(t_1) - \overline{\mu}); \tag{18}$$

3. *For any specified $\underline{\sigma}, \overline{\sigma} \in \mathbb{R}$, if $(\eta(t), \zeta(t)) \in S(\underline{\sigma}, \overline{\sigma})$ for $t \in [0, T]$, then there exist $\underline{\beta}_1, \overline{\beta}_1 \in \mathcal{IL}$ such that*

$$-\underline{\beta}_1(\underline{\sigma} - \eta(0), t) + \underline{\sigma} \leq \eta(t) \leq \overline{\beta}_1(\eta(0) - \overline{\sigma}, t) + \overline{\sigma} \tag{19}$$

   for all $t \in [0, T]$;

4. *For any specified compact $M \subset \mathbb{R}$, there exist $\underline{\beta}_2, \overline{\beta}_2 \in \mathcal{IL}$ such that if $(\eta(0), \zeta(0)) \in S(\underline{\mu}_0, \overline{\mu}_0)$ with $\underline{\mu}_0 \leq \overline{\mu}_0$ belonging to M, then one can find $\underline{\mu}(t), \overline{\mu}(t)$ satisfying*

$$-\underline{\beta}_2(\underline{\upsilon} - \underline{\mu}_0, t) + \underline{\upsilon} \leq \underline{\mu}(t) \leq \overline{\mu}(t) \leq \overline{\beta}_2(\overline{\mu}_0 - \overline{\upsilon}, t) + \overline{\upsilon} \tag{20}$$

   such that

$$(\eta(t), \zeta(t)) \in S(\underline{\mu}(t), \overline{\mu}(t)) \tag{21}$$

   for all $t \geq 0$.

The proof of Proposition 1 is given in B. The functions $\zeta = \underline{\psi}(\eta - \underline{v})$ and $\zeta = \overline{\psi}(\eta - \overline{v})$, which are used to define the invariant set $S(\underline{v}, \overline{v})$ are shown in Figure 1. Property 2 in Proposition 1 means that for any initial state $(\eta(0), \zeta(0))$, it takes a finite time t_1 such that $(\eta(t_1), \zeta(t_1))$ is inside the region such that $\underline{\psi}(\eta(t_1) - \underline{\mu}) \le \zeta(t_1) \le \overline{\psi}(\eta(t_1) - \overline{\mu})$ with some constants $\underline{\mu} \le \overline{\mu}$. It should be noted that if $\lim_{r\to-\infty} \overline{\psi}(r) = \infty$, then there is a constant $\overline{\mu}$ such that the second inequality in (18) holds with $t_1 = 0$, and if $\lim_{r\to\infty} \underline{\psi}(r) = -\infty$, then there is a constant $\underline{\mu}$ such that the first inequality in (18) holds with $t_1 = 0$. Property 3 provides estimates of upper and lower bounds of $\eta(t)$ when $(\eta(t), \zeta(t))$ is within the region $S(\underline{\sigma}, \overline{\sigma})$. By using $S(\underline{\mu}(t), \overline{\mu}(t))$, property 4 gives an estimate on the region which restricts the motion of $(\eta(t), \zeta(t))$.

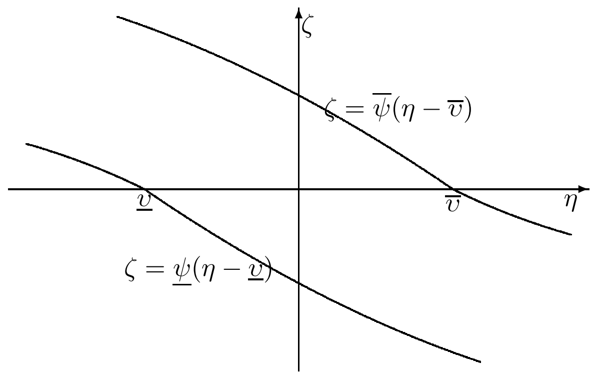

Figure 1: The curves $\zeta = \underline{\psi}(\eta - \underline{v})$ and $\zeta = \overline{\psi}(\eta - \overline{v})$ describes the invariant set $S(\underline{v}, \overline{v})$ indicated in Property 1 in Proposition 1.

Remark 1 *Condition* (17) *means "rate limit" on the nonlinear function* ϕ. *This issue has been studied in the context of system analysis via integral quadratic constraints (IQC); see e.g., [38]. However, it should be noted that the nonlinear function* φ *may not have a "rate limit". For the invariant set properties of the nonlinear system, IQC methods are not used in the analysis.*

3.2 Main Results of Output Agreement

Consider the multi-agent system (1)–(2). We propose a class of distributed control laws in the form of (3)–(4) as

$$\mu_i = \varphi_i(\zeta_i - \phi_i(\xi_i)) \tag{22}$$

$$\xi_i = \frac{1}{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}} \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\eta_i - \eta_j) \tag{23}$$

where $\sigma : [0, \infty) \to \mathcal{P}$ is a piecewise constant switching signal describing the information exchange between the systems with $\mathcal{P} \subset \mathbb{N}$ being a finite set representing all the possible information exchange topologies, $\mathcal{N}_i(p) \subseteq \{1, \ldots, N\}$

denotes the neighbor set of agent i for each $i = 1, \dots, N$ and each $p \in \mathcal{P}$, constant $a_{ij} > 0$ if $i \neq j$ and $a_{ij} \geq 0$ if $i = j$, and $\varphi_i, \phi_i : \mathbb{R} \to \mathbb{R}$ are nonincreasing, locally Lipschitz functions and satisfy

$$\varphi_i(0) = \phi_i(0) = 0, \tag{24}$$

$$\varphi_i(r)r < 0, \quad \phi_i(r)r < 0 \quad \text{for} \quad r \neq 0, \tag{25}$$

$$\sup_{r\in\mathbb{R}}\{\max \partial\varphi_i(r)\} < 4 \inf_{r\in\mathbb{R}}\{\min \partial\phi_i(r)\}, \tag{26}$$

for $i = 1, \dots, N$. It can be observed that each controlled agent (1)–(2) with control law (22)–(23) is in the form of (13)–(14), and conditions (24)–(26) are in accordance with conditions (15)–(17), by defining $\upsilon_i = \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}\eta_j / \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}$.

Each (η_i, ζ_i)-system (1)–(2) with μ_i defined in (22)–(23) can be rewritten in the form of (13)–(14):

$$\dot{\eta}_i = \zeta_i \tag{27}$$

$$\dot{\zeta}_i = \varphi_i(\zeta_i - \phi_i(\eta_i - \upsilon_i)) \tag{28}$$

by defining

$$\upsilon_i = \frac{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}\eta_j}{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}}, \tag{29}$$

which represents the influence of other agents. With the properties given by Proposition 1, we will analyze how the agents interact with each other during the distributed control process.

Before proposing the main result on strong output agreement, a switching digraph $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$ is used to model the information exchange topology between the agents, where $\mathcal{N}$ is the set of N vertices corresponding to the agents, and for each $p \in \mathcal{P}$, if $j \in \mathcal{N}_i(p)$, then there is a directed edge (j, i) belonging to $\mathcal{E}(p)$. By default, (i, i) for $i \in \mathcal{N}$ belong to $\mathcal{E}(p)$ for all $p \in \mathcal{P}$.

A digraph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ is quasi-strongly connected (QSC) if there exists a $c \in \mathcal{N}$ such that there is a directed path from c to i for each $i \in \mathcal{N}$; vertex c is called the center of $\mathcal{G}$. For a switching digraph $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$, we define the union digraph over $[t_1, t_2]$ as $\mathcal{G}(\sigma([t_1, t_2])) = (\mathcal{N}, \bigcup_{t\in[t_1,t_2]} \mathcal{E}(\sigma(t)))$. A switching digraph $\mathcal{G}(\sigma(t))$ with $\sigma : [0, \infty) \to \mathcal{P}$ is called uniformly quasi-strongly connected (UQSC) with time constant $T > 0$ if $\mathcal{G}(\sigma([t, t + T]))$ is QSC for all $t \geq 0$. A switching digraph $\mathcal{G}(\sigma(t))$ with $\sigma : [0, \infty) \to \mathcal{P}$ has an edge dwell time $\tau_D > 0$ if for each $t \in [0, \infty)$, for any directed edge $(i_1, i_2) \in \mathcal{E}(\sigma(t))$, there exists a $t^* \geq 0$ depending on t such that $t \in [t^*, t^* + \tau_D]$ and $(i_1, i_2) \in \mathcal{E}(\sigma(\tau))$ for $\tau \in [t^*, t^* + \tau_D]$.

Lemma 1 *Consider a switching digraph $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$ with $\sigma : [0, \infty) \to \mathcal{P}$, which is UQSC with time constant $T > 0$ and has an edge dwell time $\tau_D > 0$. If $c \in \mathcal{N}$ is a center of $\mathcal{G}(\sigma([t, t + T]))$, then for any $\mathcal{N}_1$ such that $c \in \mathcal{N}_1$, there exist $i_1 \in \mathcal{N}_1$, $i_2 \in \mathcal{N}\backslash\mathcal{N}_1$ and $t' \in [t - \tau_D, t + T]$ such that $(i_1, i_2) \in \mathcal{E}(\sigma(\tau))$ for $\tau \in [t', t' + \tau_D]$.*

Lemma 1 can be proved by directly using the definitions of UQSC and edge dwell-time.

The following theorem presents our main result on strong output agreement problem.

Theorem 1 *Consider the double-integrators* (1)–(2) *with control laws in the form of* (22)–(23)*. Assume conditions* (24)–(26) *are satisfied. If* $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$ *with* $\sigma : [0, \infty) \to \mathcal{P}$ *is UQSC and has an edge dwell-time* $\tau_D > 0$*, then the strong output agreement problem is solved.*

The proof of Theorem 1, which is based on Proposition 1, is in Section 3.3.

Remark 2 *In the previously published works on coordinated control of continuous-time systems with switching topologies, e.g., [21, 33, 50], it is usually assumed that the switching graph has a dwell-time* τ'_D*, which means each specific topology remains unchanged for a period larger than* τ'_D*. As edge dwell-time will be directly used in our analysis, we assume edge dwell-time instead of the usually used graph dwell-time. It should be noted that the assumption of edge dwell-time does not cause restrictions to the main results. In fact, the edge dwell-time is normally larger than the graph dwell-time for a specific switching graph.*

Remark 3 *Conditions* (24)–(26) *allow us to choose bounded and nonsmooth* ϕ_i *for* $i = 1, \ldots, N$*. One example of* ϕ_i *is*

$$\phi_i(r) = \begin{cases} -1, & \textit{when } r > 1; \\ r, & \textit{when } -1 \le r \le 1; \\ 1, & \textit{when } r < -1. \end{cases}$$

Correspondingly, we can choose $\varphi_i(r) = -kr$ *with constant* $k > 4$*. This could be of practical interest when the velocities* ζ_i *are required to be bounded in the process of controlling the positions* η_i *to achieve agreement. Consider the* ζ_i*-system* (2) *with* μ_i *defined by* (22)*. With bounded* ϕ_i*, the velocity* ζ_i *can be restricted to within a specific bounded range depending on the initial state* $\zeta_i(0)$ *and the bounds of* ϕ_i*. With bounded velocities, we can also guarantee the boundedness of the control signals* μ_i*, which may be required to be bounded due to actuator saturation. It should also be noted that for the agents modelled by double-integrators, even if the control (actuator) signals are saturated, the velocities may not be saturated.*

Remark 4 *This paper assumes the availability of the velocities* ζ_i *of the agents. If* ζ_i *is not available, then an observer-based distributed control design would be of interest. Such a result can be found in [18]. However, it needs more efforts to deal with the problems caused by physical constraints (e.g., velocity saturation) and communication constraints (e.g., switching topology) with an observer-based design.*

In a leader-follower structure, the motion of the leader does not depend on the outputs of the followers and the output of the leader is accessible to some of the followers. Based on Theorem 1, we have the following corollary for output agreement of multi-agent systems with a leader.

Corollary 1 *For a group of systems with a leader i^*, to achieve strong output agreement, it is required that there exists a finite constant $T > 0$ such that for all $t \geq 0$, the union digraph $\mathcal{G}(\sigma([t, t+T]))$ is QSC with i^* as a center.*

Remark 5 *It is well-known that the leader-follower structure is fragile with respect to the failure of the leader. It is essential to retain necessary connectivity in case the leader fails. Since our design does not assume tree topology, one possible way to improve the system reliability is to employ candidate leaders and introduce circles to the information exchange digraph. Specifically, the leader and the candidates are arranged on a circle in the information exchange digraph, so that when the original leader fails and changes its role to follower, the leadership can be passed to one of the candidates and the connectivity condition is still satisfied. Note that this strategy is also valid for the formation control studied in Section 4.*

Remark 6 *Intuitively, if the requirement of QSC is not fulfilled, then there must be at least two agents who are not influenced by some common agent, directly or indirectly, and thus agreement may not be expectable. However, based on the proof of Theorem 1, we can still guarantee the boundedness of the agents' states even if the QSC condition is not satisfied; see* (33)–(34)*. It is of practical interest to study the practical convergence under relaxed connectivity assumptions.*

3.3 Proof of Theorem 1

With conditions (24)–(26) satisfied, each closed-loop (η_i, ζ_i)-system has the properties given in Proposition 1.

According to Property 1 in Proposition 1, for $i = 1, \ldots, N$, one can find $\underline{\psi}_i, \overline{\psi}_i$ such that

$$S_i(\underline{v}_i, \overline{v}_i) = \left\{(\eta_i, \zeta_i) : \underline{\psi}_i(\eta_i - \underline{v}_i) \leq \zeta_i \leq \overline{\psi}_i(\eta_i - \overline{v})\right\}$$

is an invariant set of the (η_i, ζ_i)-system if $v_i \in [\underline{v}_i, \overline{v}_i]$.

Given the chosen $\underline{\psi}_i, \overline{\psi}_i$, we suppose that there exist $\underline{\mu}_i(0), \overline{\mu}_i(0)$ such that

$$(\eta_i(0), \zeta_i(0)) \in S_i(\underline{\mu}_i(0), \overline{\mu}_i(0)) \tag{30}$$

$$\underline{\mu}_i(0) \leq \eta_i(0) \leq \overline{\mu}_i(0) \tag{31}$$

for $i \in \mathcal{N}$. Otherwise, there exists a finite time t^*, at which property (30) holds, according to Property 2 in Proposition 1. Here, we use $\underline{\mu}_i$ and $\overline{\mu}_i$ together with S_i to represent the region of motion of η_i, ζ_i.

The basic idea of the proof is to find appropriate $\underline{\mu}_i(t), \overline{\mu}_i(t)$ such that

$$\underline{\psi}_i(\eta_i(t) - \underline{\mu}_i(t)) \leq \zeta_i(t) \leq \overline{\psi}_i(\eta_i(t) - \overline{\mu}_i(t)). \tag{32}$$

We first show the boundedness of the signals. Define $\underline{\mu} = \min_{i\in\mathcal{N}} \underline{\mu}_i$ and $\overline{\mu} = \max_{i\in\mathcal{N}} \overline{\mu}_i$. Denote $\eta = [\eta_1, \ldots, \eta_N]^T$ and $\zeta = [\zeta_1, \ldots, \zeta_N]^T$. Then, $(\eta(0), \zeta(0)) \in S(\underline{\mu}(0), \overline{\mu}(0))$ with

$$S(\underline{\mu}(0), \overline{\mu}(0)) = \{(\eta, \zeta) : (\eta_i, \zeta_i) \in S_i(\underline{\mu}(0), \overline{\mu}(0)) \text{ for } i \in \mathcal{N}\}.$$

For each $i \in \mathcal{N}$, (29) implies $\underline{\mu}(0) \leq \upsilon_i \leq \overline{\mu}(0)$ if $\underline{\mu}(0) \leq \eta_i \leq \overline{\mu}(0)$. Based on Proposition 1, it can be proved that $S(\underline{\mu}(0), \overline{\mu}(0))$ is an invariant set of the interconnected system with (η, ζ) as the state. Thus,

$$(\eta(t), \zeta(t)) \in S(\underline{\mu}(0), \overline{\mu}(0)) \tag{33}$$

$$\underline{\mu}(0) \leq \upsilon_i(t) \leq \overline{\mu}(0) \tag{34}$$

for all $t \geq 0$. Note that the proof of (33) and (34) does not use the connectivity condition.

To prove the output agreement, we define two sets, $\mathcal{Q}_1$ and $\mathcal{Q}_2$, which satisfy $\mathcal{Q}_1 \cup \mathcal{Q}_2 = \mathcal{N}$ and have the following properties: if $i \in \mathcal{Q}_1$, then η_i satisfies

$$\eta_i(0) \geq (\underline{\mu}(0) + \overline{\mu}(0))/2; \tag{35}$$

if $i \in \mathcal{Q}_2$, then $\eta_i(0)$ satisfies

$$\eta_i(0) \leq (\underline{\mu}(0) + \overline{\mu}(0))/2. \tag{36}$$

Note that either $\mathcal{Q}_1$ or $\mathcal{Q}_2$ can be an empty set. Also, the existence of the pair $(\mathcal{Q}_1, \mathcal{Q}_2)$ may not be unique.

Denote N_2 as the size of set $\mathcal{Q}_2$. Define $T^* = N_2(T + 2\tau_D + \Delta_T) + \Delta_T$ with $\Delta_T > 0$. For each $i \in \mathcal{Q}_1$, with Property 3 in Proposition 1, there exist $\underline{\beta}_{i1} \in \mathcal{IL}$ such that

$$\begin{aligned}
\eta_i(t) &\geq -\underline{\beta}_{i1}(\underline{\mu}(0) - \eta_i(0), t) + \underline{\mu}(0) \\
&\geq -\underline{\beta}_{i1}\left(\frac{\underline{\mu}(0) - \overline{\mu}(0)}{2}, t\right) + \underline{\mu}(0) \\
&\geq -\underline{\beta}_{i1}\left(\frac{\underline{\mu}(0) - \overline{\mu}(0)}{2}, T^*\right) + \underline{\mu}(0) \\
&:= \breve{\alpha}_i(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0),
\end{aligned} \tag{37}$$

holds for all $t \in [0, T^*]$, where it can be directly checked that $\breve{\alpha}_i$ is a continuous, positive definite function and satisfies $\breve{\alpha}_i < \text{Id}$.

Denote i^* as the center of the union digraph $\mathcal{G}(\sigma([0, T^*]))$, and without loss of generality, suppose that $i^* \in \mathcal{Q}_1$. Recursively define set $\mathcal{F}_k = \{f_1, \ldots, f_k\}$ for $k = 1, \ldots, N_2$ such that

- there exists an $l_1 \in \mathcal{Q}_1$ such that $(l_1, f_1) \in \mathcal{E}(\sigma([t', t' + \tau_D]))$ with $[t', t' + \tau_D] \subseteq [0, T + 2\tau_D]$;

- for $k = 2, \ldots, N_2$, there exists an $l_k \in \mathcal{Q}_1 \cup \mathcal{F}_{k-1}$ such that $(l_k, f_k) \in \mathcal{E}(\sigma([t', t'+\tau_D]))$ with $[t', t'+\tau_D] \subseteq [(k-1)(T+2\tau_D+\Delta_T), k(T+2\tau_D+\Delta_T)-\Delta_T]$.

The existence of such f_k is guaranteed by Lemma 1. For convenience of notations, denote $\mathcal{F}_0 = \emptyset$.

In the following procedure, we first prove the validity of property (37) for $i \in \mathcal{Q}_2$ and thus for all $i \in \mathcal{N}$. Then, we find appropriate $\underline{\mu}$ and $\overline{\mu}$ for $S(\underline{\mu}, \overline{\mu})$ so that properties (30) and (31) always hold and at the same time $\underline{\mu}$ and $\overline{\mu}$ converge to each other.

We now prove property (37) for $i \in \mathcal{Q}_2$. Note that (37) holds for $i \in \mathcal{Q}_1$ for all $t \in [0, T^*]$. Suppose that (37) holds for $i \in \mathcal{Q}_1 \cup \mathcal{F}_{k-1}$ for all $t \in [(k-1)(T+2\tau_D+\Delta_T), T^*]$. We study the motion of different variables of the f_k-subsystem during the following time intervals:

1) $t \in [t', t'+\tau_D] \subseteq [(k-1)(T+2\tau_D+\Delta_T), k(T+2\tau_D+\Delta_T)-\Delta_T]$. During this time interval, we have

$$\begin{aligned}
\upsilon_{f_k}(t) &= \frac{\sum_{j\in\mathcal{N}_{f_k}(\sigma(t))} a_{f_k j}\eta_j(t)}{\sum_{j\in\mathcal{N}_{f_k}(\sigma(t))} a_{f_k j}} \\
&= \frac{\sum_{j\in\mathcal{N}_{f_k}(\sigma(t))\setminus\{l_k\}} a_{f_k j}\eta_j(t) + a_{f_k l_k}\eta_{l_k}(t)}{\sum_{j\in\mathcal{N}_{f_k}(\sigma(t))} a_{f_k j}} \\
&\geq \frac{a_{f_k l_k}\breve{\alpha}_{l_k}(\overline{\mu}(0)-\underline{\mu}(0))}{\sum_{j\in\mathcal{N}_{f_k}(\sigma(t))} a_{f_k j}} + \underline{\mu}(0) \\
&:= \hat{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)) + \underline{\mu}(0)
\end{aligned}$$

where $\hat{\alpha}_{f_k}$ is continuous, positive definite and less than Id.

Then, by using property 4 of Proposition 1, we have

$$(\eta_{f_k}(t), \zeta_{f_k}(t)) \in S_{f_k}(\underline{\mu}'_{f_k}(t), \overline{\mu}(0)) \tag{38}$$

holds with

$$\begin{aligned}
\underline{\mu}'_{f_k}(t) = &-\underline{\beta}_{f_k 2}(\hat{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)), t-t') \\
&+ \hat{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)) + \underline{\mu}(0)
\end{aligned}$$

where $\underline{\beta}_{f_k 2} \in \mathcal{IL}$. Thus,

$$\begin{aligned}
\underline{\mu}'_{f_k}(t'+\tau_D) = &-\underline{\beta}_{f_k 2}(\hat{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)), \tau_D) \\
&+ \hat{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)) + \underline{\mu}(0) \\
:= &\ \bar{\alpha}_{f_k}(\overline{\mu}(0)-\underline{\mu}(0)) + \underline{\mu}(0),
\end{aligned}$$

where it can be verified that $\bar{\alpha}_{f_k}$ is continuous, positive definite and less than Id.

2) $t \in [t' + \tau_D, T^*]$. During this time interval,

$$\upsilon_{f_k}(t) \geq \underline{\mu}(0).$$

By using property 4 of Proposition 1 again, we have that property (38) holds with

$$\begin{aligned}\underline{\mu}'_{f_k}(t) &= -\underline{\beta}_{f_k 2}(\bar{\alpha}_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)), t - t' - \tau_D) \\ &\quad + \bar{\alpha}_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0) \\ &\geq -\underline{\beta}_{f_k 2}(\bar{\alpha}_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)), T^*) \\ &\quad + \bar{\alpha}_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0) \\ &:= \bar{\alpha}'_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0).\end{aligned}$$

3) $t \in [k(T + 2\tau_D + \Delta_T), T^*]$. During this time interval, by using property 4 of Proposition 1, we have

$$\begin{aligned}\eta_{f_k}(t) &\geq -\underline{\beta}_{f_k 1}(\bar{\alpha}'_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)), \Delta_T) \\ &\quad + \bar{\alpha}'_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0) \\ &:= \breve{\alpha}_{f_k}(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0).\end{aligned}$$

By recursively studying the cases with $k = 1, \ldots, N_2$, it can be proved that property (37) holds for all $i \in \mathcal{N}$ for $t \in [N_2(T + 2\tau_D + \Delta_T), T^*]$.

Based on the achievement at Step 3), following a similar reasoning as in Steps 1) and 2) for f_k, it can also be proved that

$$(\eta_i(T^*), \zeta_i(T^*)) \in S_i(\underline{\mu}'_i(T^*), \overline{\mu}(0))$$

holds for all $i \in \mathcal{N}$, with

$$\underline{\mu}'_i(T^*) \geq \bar{\alpha}'_i(\overline{\mu}(0) - \underline{\mu}(0)) + \underline{\mu}(0)$$

where $\bar{\alpha}'_i$ is continuous, positive definite and less than Id.

Define

$$\begin{aligned}&\underline{\mu}(T^*) = \min_{i \in \mathcal{N}}\{\bar{\alpha}'_i(\overline{\mu}(0) - \underline{\mu}(0)), \breve{\alpha}_i(\overline{\mu}(0) - \underline{\mu}(0))\} + \underline{\mu}(0), \\ &\overline{\mu}(T^*) = \overline{\mu}(0).\end{aligned}$$

Define $\tilde{\mu} = \overline{\mu} - \underline{\mu}$. Then, there exists a continuous and positive definite $\tilde{\alpha} < \text{Id}$ such that

$$\tilde{\mu}(T^*) = \overline{\mu}(T^*) - \underline{\mu}(T^*) \leq \tilde{\mu}(0) - \tilde{\alpha}(\tilde{\mu}(0)).$$

In this proof, we only considered the case of $i^* \in \mathcal{Q}_1$, the case of $i^* \in \mathcal{Q}_2$ can be studied in the same way due to symmetry.

By recursively analyzing the system we can achieve

$$\tilde{\mu}((k+1)T^*) \leq \tilde{\mu}(kT^*) - \tilde{\alpha}(\tilde{\mu}(kT^*))$$

for $k \in \mathbb{Z}_+$. By using the asymptotic stability result for discrete-time nonlinear systems in [24], we can conclude that $\tilde{\mu}(kT^*) \to 0$ as $k \to \infty$.

Define $\overline{\mu}(t) = \overline{\mu}(kT^*)$ and $\underline{\mu}(t) = \underline{\mu}(kT^*)$ if $t \in [kT^*, (k+1)T^*)$. According to the analysis above, during the control procedure, it always holds that

$$(\eta_i(t), \zeta_i(t)) \in S_i(\underline{\mu}(t), \overline{\mu}(t))$$

and

$$\underline{\mu}(t) \leq \eta_i(t) \leq \overline{\mu}(t).$$

Properties (5) and (6) can be proved as $\tilde{\mu} = \overline{\mu} - \underline{\mu}$ asymptotically converges to the origin. This ends the proof of Theorem 1.

4 Distributed Formation Control of Mobile Robots

As an application of the main result in Section 3, in this section, we propose a class of distributed control laws for formation control of a group of $N+1$ mobile robots (7)–(9) with $i = 0, \ldots, N$ with flexible topologies. Different from our recent paper [35], the position sensing topology of the mobile robot system studied in this section is allowed to be time-variable.

The following assumption is made throughout this section.

Assumption 1 *The linear velocity v_0 of the leader robot is differentiable with bounded derivative, i.e., $\dot{v}_0(t)$ exists and is bounded on $[0, \infty)$, and there exists constants $\overline{v}_0 > \underline{v}_0 > 0$ such that $\underline{v}_0 \leq v_0(t) \leq \overline{v}_0$ for all $t \geq 0$.*

Global position measurements of the robots are usually unavailable and the sensing topology may be switching in practical formation control systems. We will employ the strong output agreement result proposed in the previous section to develop a new class of coordinated controllers, which are capable of overcoming the problems caused by nonholonomic constraint and achieving the formation control objective by using local relative position measurements under switching position sensing topologies as well as the velocity information of the leader. With our new coordinated controller, the velocity v_i of each i-th follower robots can also be guaranteed to be upper bounded by a constant $\lambda^U > \overline{v}_0$ if required.

In our design, we assume the availability of the velocity information v_0, θ_0, ω_0 of the leader robot to the follower robots. This could be realizable if the communication capability is strong enough. If each follower robot also has access to the real-time relative position information $(x_i - x_0, y_i - y_0)$, then distributed control is not needed. For this purpose, one may consider calculating $(x_i - x_0, y_i - y_0)$ by exchanging the information of relative positions $(x_i - x_j, y_i - y_j)$ measured by the agents. However, due to the switching sensing topology, this may not be realizable unless some more restrictive connectivity condition is satisfied by the position sensing topology.

The distributed controller for each follower robot is composed of two stages: (a) Initialization; (b) Formation control. With the initialization stage, the heading direction of each follower robot can be controlled to converge to within desired ranges in some finite time. Then, the formation control stage is triggered and the formation control objective is achieved during the formation control stage.

4.1 Initialization Stage

The objective of the initialization stage is to control the angles $\theta_i(t)$ for $i = 1, \ldots, N$ to within a specific small neighborhood of $\theta_0(t)$.

For each i-th mobile robot (7)–(9), we propose the following initialization control law:

$$\omega_i = \phi_{\theta i}(\theta_i - \theta_0) + \omega_0 \tag{39}$$

$$v_i = v_0 \tag{40}$$

where $\phi_{\theta i} : \mathbb{R} \to \mathbb{R}$ is a nonlinear function such that $\phi_{\theta i}(r)r < 0$ for $r \neq 0$ and $\phi_{\theta i}(0) = 0$.

Define $\tilde{\theta}_i = \theta_i - \theta_0$. By taking the derivative of $\tilde{\theta}_i$ and using (39) and (9), we have

$$\dot{\tilde{\theta}}_i = \phi_{\theta i}(\tilde{\theta}_i). \tag{41}$$

With the appropriately designed $\phi_{\theta i}$, we can guarantee the asymptotic stability of system (41). Moreover, there exists $\beta_{\tilde{\theta}} \in \mathcal{KL}$ such that $|\tilde{\theta}(t)| \leq \beta_{\tilde{\theta}}(|\tilde{\theta}(0)|, t)$ for $t \geq 0$.

For specified $0 < \lambda^L < \underline{v}_0 < \overline{v}_0 < \lambda^U$, define

$$\lambda = \min\left\{ \frac{\sqrt{2}}{2}(\underline{v}_0 - \lambda^L), \frac{\sqrt{2}}{2}(\lambda^U - \overline{v}_0) \right\}. \tag{42}$$

By directly using the property of continuous functions, there exists a $\overline{\delta}_{\theta 0} > 0$ such that

$$|v_0 \cos(\theta_0 + \delta_{\theta 0}) - v_0 \cos\theta_0| \leq \lambda,$$
$$|v_0 \sin(\theta_0 + \delta_{\theta 0}) - v_0 \sin\theta_0| \leq \lambda$$

for all $v_0 \in [\underline{v}_0, \overline{v}_0]$, $\theta_0 \in \mathbb{R}$ and $|\delta_{\theta 0}| \leq \overline{\delta}_{\theta 0}$. Recall that for any $\beta \in \mathcal{KL}$, there exist $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ such that $\beta(s,t) \leq \alpha_1(s)\alpha_2(e^{-t})$ for all $s, t \in \mathbb{R}_+$ [51, Lemma

8]. With control law (39)–(40), there exists a finite time T_{Oi} for the i-th robot such that $|\theta_i(T_{Oi}) - \theta_0(T_{Oi})| \leq \overline{\delta}_{\theta 0}$, and thus,

$$|v_i(T_{Oi})\cos\theta_i(T_{Oi}) - v_0(T_{Oi})\cos\theta_0(T_{Oi})| \leq \lambda, \tag{43}$$

$$|v_i(T_{Oi})\sin\theta_i(T_{Oi}) - v_0(T_{Oi})\sin\theta_0(T_{Oi})| \leq \lambda. \tag{44}$$

At time T_{Oi}, the distributed control law for the i-th follower robot is switched to the formation control stage, as demonstrated below.

Remark 7 *For the initialization stage, the accessibility of the linear velocity and heading direction of the leader means the availability of a global reference system to the follower robots. This allows us to design non-distributed control laws. It should be noted that the heading directions are defined in a non-Euclidean space. In case some distributed protocol is needed, one may have to face the agreement problem in non-Euclidean spaces. This problem is outside the scope of this paper but of practical interest when the leader's information is unavailable. The interested reader may consult [29, 48, 3] for some recent results.*

4.2 Formation Control Stage

As shown in [7, 13], the unicycle model (7)–(9) is dynamic feedback linearizable. For each $i = 1, \ldots, N$, we introduce a new input $r_i \in \mathbb{R}$ such that

$$\dot{v}_i = r_i. \tag{45}$$

Define $v_{xi} = v_i \cos\theta_i$ and $v_{yi} = v_i \sin\theta_i$. Then, $\dot{x}_i = v_{xi}$ and $\dot{y}_i = v_{yi}$. By taking the derivative of v_{xi} and v_{yi}, respectively, we have

$$\begin{pmatrix} \dot{v}_{xi} \\ \dot{v}_{yi} \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -v_i\sin\theta_i \\ \sin\theta_i & v_i\cos\theta_i \end{pmatrix} \begin{pmatrix} r_i \\ \omega_i \end{pmatrix}$$

In the case of $v_i \neq 0$, by designing

$$\begin{pmatrix} r_i \\ \omega_i \end{pmatrix} = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\frac{\sin\theta_i}{v_i} & \frac{\cos\theta_i}{v_i} \end{pmatrix} \begin{pmatrix} u_{xi} \\ u_{yi} \end{pmatrix} \tag{46}$$

we can transform unicycle model (7)–(9) into two double-integrators with new control inputs u_{xi} and u_{yi}:

$$\dot{x}_i = v_{xi}, \quad \dot{v}_{xi} = u_{xi}, \tag{47}$$

$$\dot{y}_i = v_{yi}, \quad \dot{v}_{yi} = u_{yi}. \tag{48}$$

Define $\tilde{x}_i = x_i - x_0 - d_{xi0}$, $\tilde{y}_i = y_i - y_0 - d_{yi0}$, $\tilde{v}_{xi} = v_{xi} - v_{x0}$, $\tilde{v}_{yi} = v_{yi} - v_{y0}$, $\tilde{u}_{xi} = u_{xi} - u_{x0}$, and $\tilde{u}_{yi} = u_{yi} - u_{y0}$. Then,

$$\dot{\tilde{x}}_i = \tilde{v}_{xi}, \quad \dot{\tilde{v}}_{xi} = \tilde{u}_{xi}, \tag{49}$$

$$\dot{\tilde{y}}_i = \tilde{v}_{yi}, \quad \dot{\tilde{v}}_{yi} = \tilde{u}_{yi}. \tag{50}$$

In this way, the formation control problem is transformed into the issue of designing control laws for system (49)–(50) with $\tilde{u}_{xi}$ and $\tilde{u}_{yi}$ as the control inputs, so that $v_i \neq 0$ is guaranteed, and at the same time, the formation control objective is achieved.

Remark 8 *The transformation defined in* (46) *is valid only if* $v_i \neq 0$*; otherwise, the transformation from* (u_{xi}, u_{yi}) *to* (r_i, ω_i) *may be singular in the process of feedback control. This is basically caused by the nonholonomic constraint of the unicycle model [7]. The requirement of* $v_i \neq 0$ *means restrictions on* $\tilde{v}_{xi}$ *and* $\tilde{v}_{yi}$*, which has not been paid enough attention to in the previously published papers involving distributed control of double-integrators, e.g., [54, 43, 2, 59, 46, 44].*

The condition $v_i \neq 0$ for the validity of (47)–(48) can be equivalently represented by $\sqrt{v_{xi}^2 + v_{yi}^2} > 0$ based on the definition of v_{xi} and v_{yi}. To implement the transformation in (46), we need to design the control law for the i-th robot such that

$$\max\{|\tilde{v}_{xi}|, |\tilde{v}_{yi}|\} \leq \frac{\sqrt{2}}{2}(\underline{v}_0 - \lambda^L) \tag{51}$$

for a specified $0 < \lambda^L < \underline{v}_0$. In doing so, we can guarantee that $|v_i| = \sqrt{v_{xi}^2 + v_{yi}^2} = \sqrt{(v_{x0} + \tilde{v}_{xi})^2 + (v_{y0} + \tilde{v}_{yi})^2} \geq \lambda^L > 0$ and thus $v_i \neq 0$.

Similarly, to guarantee that $|v_i| \leq \lambda^U$ for any given $\lambda^U > \overline{v}_0$, we will design a control law such that

$$\max\{|\tilde{v}_{xi}|, |\tilde{v}_{yi}|\} \leq \frac{\sqrt{2}}{2}(\lambda^U - \overline{v}_0). \tag{52}$$

The formation control problem is now transformed into the issue of designing control laws for system (49)–(50) with $\tilde{u}_{xi}$ and $\tilde{u}_{yi}$ as the control inputs, so that (51) and (52) are guaranteed during the control procedure, and at the same time, the formation control objective is achieved.

Recall the definition of λ in (42). Both (51) and (52) can be satisfied if

$$\max\{|\tilde{v}_{xi}|, |\tilde{v}_{yi}|\} \leq \lambda. \tag{53}$$

After the initialization stage, at time T_{Oi}, the satisfaction of (43) and (44) implies that (53) is satisfied at time T_{Oi}.

Considering the requirement of relative position measurement, we propose a distributed control law in the following form:

$$\tilde{u}_{xi} = \varphi_{xi}(\tilde{v}_{xi} - \phi_{xi}(z_{xi})) \tag{54}$$

$$\tilde{u}_{yi} = \varphi_{yi}(\tilde{v}_{yi} - \phi_{yi}(z_{yi})) \tag{55}$$

where $\varphi_{xi}, \varphi_{yi}, \phi_{xi}, \phi_{yi}$ are strictly decreasing and locally Lipschitz, and satisfy $\varphi_{xi}(0) = \varphi_{yi}(0) = \phi_{xi}(0) = \phi_{yi}(0) = 0$, $\varphi_{xi}(r)r < 0$, $\varphi_{yi}(r)r < 0$, $\phi_{xi}(r)r < 0$

and $\phi_{yi}(r)r < 0$ for $r \neq 0$, and

$$\sup_{r\in\mathbb{R}}\{\max \partial\varphi_{xi}(r) : r \in \mathbb{R}\} < 4 \inf_{r\in\mathbb{R}}\{\min \partial\phi_{xi}(r)\}, \tag{56}$$

$$\sup_{r\in\mathbb{R}}\{\max \partial\varphi_{yi}(r) : r \in \mathbb{R}\} < 4 \inf_{r\in\mathbb{R}}\{\min \partial\phi_{yi}(r)\}. \tag{57}$$

for $i = 1, \ldots, N$. The functions ϕ_{xi}, ϕ_{yi} are also designed to satisfy

$$-\lambda \leq \phi_{xi}(r), \phi_{yi}(r) \leq \lambda \tag{58}$$

for $r \in \mathbb{R}$.

The variables z_{xi} and z_{yi} are defined as

$$z_{xi} = \frac{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(x_i - x_j - d_{xij})}{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}} \tag{59}$$

$$z_{yi} = \frac{\sum_{j\in\mathcal{N}_i(\sigma(t))} b_{ij}(y_i - y_j - d_{yij})}{\sum_{j\in\mathcal{N}_i(\sigma(t))} b_{ij}} \tag{60}$$

where $\sigma : [0, \infty) \to \mathcal{P}$ is a piecewise constant switching signal describing the position sensing topology with $\mathcal{P}$ being the set of all the possible position sensing topologies, $\mathcal{N}_i(p) \subseteq \{0, \ldots, N\}$ for each $i = 1, \ldots, N$ and each $p \in \mathcal{P}$, constant $a_{ij} > 0$ if $i \neq j$ and $a_{ij} \geq 0$ if $i = j$. Note that d_{xij}, d_{yij} in (59) and (60) represent the desired relative position between the i-th robot and the j-th robot. By default, $d_{xii} = d_{yii} = 0$.

Consider the $(\tilde{v}_{xi}, \tilde{v}_{yi})$-system defined in (49)–(50). With (43) and (44) achieved, the boundedness of ϕ_{xi} and ϕ_{yi} in (58) together with the control law (54)–(55) guarantees that

$$\max\{|\tilde{v}_{xi}(t)|, |\tilde{v}_{yi}(t)|\} \leq \lambda \tag{61}$$

for $t \geq T_{Oi}$. For the proof of (61), we can consider $\{(\tilde{v}_{xi}, \tilde{v}_{yi}) : \max\{|\tilde{v}_{xi}|, |\tilde{v}_{yi}|\} \leq \lambda\}$ as an invariant set of the $(\tilde{v}_{xi}, \tilde{v}_{yi})$-system. With control law (54), the $\tilde{v}_{xi}$-subsystem can be rewritten as $\tilde{v}_{xi} = \varphi_{xi}(\tilde{v}_{xi} - \phi_{xi}(z_{xi}))$. Note that $\phi_{xi}(r) \in [-\lambda, \lambda]$ for all $r \in \mathbb{R}$ and $\varphi_{xi}(r)r < 0$. It can be verified that $\dot{\tilde{v}}_{xi} \leq \varphi_{xi}(\tilde{v}_{xi} - \lambda)$ if $\tilde{v}_{xi} \geq \lambda$ and $\dot{\tilde{v}}_{xi} \geq \varphi_{xi}(\tilde{v}_{xi} + \lambda)$ if $\tilde{v}_{xi} \leq -\lambda$. This shows the invariance of $|\tilde{v}_{xi}| \leq \lambda$ for the $\tilde{v}_{xi}$-subsystem. With (61) achieved, we have $\max_{i=1,\ldots,N}\{|\tilde{v}_{xi}(t)|, |\tilde{v}_{yi}(t)|\} \leq \lambda$ for all $t \geq T_O$ with $T_O := \max_{i=1,\ldots,N}\{T_{Oi}\}$. This guarantees the validity of the transformed model (49)–(50).

Consider the multi-robot model (7)–(9) and the distributed control laws defined by (39), (40), (45), (46), (54) and (55) with nonlinear functions $\varphi_{xi}, \varphi_{yi}, \phi_{xi}, \phi_{yi}$ satisfying (56), (57) and (58).

We represent the switching position sensing topology by a switching digraph $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$, where $\mathcal{N} = \{0, \ldots, N\}$ and $\mathcal{E}(\sigma(t))$ is defined based on $\mathcal{N}_i(\sigma(t))$ given in (59) and (60) for $i = 1, \ldots, N$ and $\mathcal{N}_0(\sigma(t)) \equiv \{0\}$. Theorem 2 presents our main result on formation control of unicycle mobile robots.

Theorem 2 *Under Assumption 1, if $\mathcal{G}(\sigma(t)) = (\mathcal{N}, \mathcal{E}(\sigma(t)))$ with $\sigma : [0, \infty) \to \mathcal{P}$ is UQSC and has an edge dwell-time $\tau_D > 0$, then (10)–(12) is achieved for any $i, j = 0, \ldots, N$. Moreover, given any $\lambda^U > \overline{v}_0$, if $v_i(0) \leq \lambda^U$ for $i = 1, \ldots, N$, then $v_i(t) \leq \lambda^U$ for all $t \geq 0$.*

Proof. The states of the mobile robots remain bounded during the finite time interval $[0, T_O]$. We study the motion of the robots during $[T_O, \infty)$. Note that the model (47)–(48) is valid during $[T_O, \infty)$.

Note that $\tilde{v}_{x0} = \tilde{v}_{y0} = \tilde{u}_{x0} = \tilde{u}_{y0} = 0$. One can find appropriate $\varphi_{x0}, \varphi_{y0}, \phi_{x0}, \phi_{y0}, a_{00}, b_{00}$ to represent $\tilde{u}_{x0}$ and $\tilde{u}_{y0}$ by (54) and (55) with z_{x0} and x_{y0} in the form of (59) and (60), respectively.

For $i = 0, \ldots, N$, rewrite

$$z_{xi} = \frac{\sum_{j \in \mathcal{N}_i(\sigma(t))} a_{ij}(\tilde{x}_i - \tilde{x}_j)}{\sum_{j \in \mathcal{N}_i(\sigma(t))} a_{ij}}, \tag{62}$$

$$z_{yi} = \frac{\sum_{j \in \mathcal{N}_i(\sigma(t))} b_{ij}(\tilde{y}_i - \tilde{y}_j)}{\sum_{j \in \mathcal{N}_i(\sigma(t))} b_{ij}}. \tag{63}$$

For $i = 0, \ldots, N$, all the $\tilde{u}_{xi}$ and $\tilde{u}_{yi}$ (defined by (54) and (55), respectively) are in the form of μ_i defined in (22) and all the z_{xi} and z_{yi} (defined by (62) and (63), respectively) are in the form of ξ_i defined in (23).

Theorem 1 guarantees that

$$\lim_{t \to \infty} (\tilde{x}_i(t) - \tilde{x}_j(t)) = 0, \tag{64}$$

$$\lim_{t \to \infty} (\tilde{y}_i(t) - \tilde{y}_j(t)) = 0. \tag{65}$$

for any $i, j = 0, \ldots, N$. Then, we can prove (10) and (11) by using the definitions of $\tilde{x}_i$ and $\tilde{y}_i$ and the fact that $\tilde{x}_0(t) = \tilde{y}_0(t) \equiv 0$. The result of $v_i(t) \leq \lambda^U$ can be proved based on the discussions below (61).

By using Theorem 1, we can also prove the convergence of $\tilde{v}_{xi}, \tilde{v}_{yi}$ to the origin. By using the definitions of $\tilde{v}_{xi}, \tilde{v}_{yi}$, the convergence such that (12) can be proved. This ends the proof. $\diamondsuit$

Remark 9 *The advantage of the design in this section lies in that the distributed formation control law for the nonholonomic mobile robots uses relative position measurements, and moreover, the position sensing topology can be time-varying. This could be of interest for the systems that do not have access to accurate global position measurements. The price paid for this is that the linear velocity and the heading direction of the leader robot are assumed to be accessible to the follower robots. The problem of relaxing the restrictiveness caused by the requirement on the leader's velocity information is left for further research.*

Remark 10 *In the case of time-varying formation geometry, the d_{xij} and d_{yij} are supposed to be time-varying. The formation control problem can be realized as long as the problem can be transformed into an output agreement problem for* *systems in the form of* (49)–(50)*, and at the same time, the linear velocities are guaranteed to be nonzero (due to nonholonomic constraint). The nonholonomic constraint still causes the major difficulty. In the case of time-varying formation geometry, condition in the form of* (51) *may not be valid any more, and some more restrictive condition should be found first.*

Remark 11 *If there is no physical leader and instead a virtual leader is employed to send reference heading direction and linear velocities to the robots, the proposed distributed control strategy can be slightly modified for formation control. In this case, one may employ a switching digraph* $\mathcal{G}^f(\sigma(t)) = (\mathcal{N}^f, \mathcal{E}^f(\sigma(t)))$ *to represent the position sensing topology of the follower robots, where* $\mathcal{N}^f = \{1, \ldots, N\}$ *and* $\mathcal{E}^f(\sigma(t))$ *is defined based on* $\mathcal{N}_i(\sigma(t))$ *given in* (59) *and* (60) *for* $i = 1, \ldots, N$*. Under Assumption 1, the formation control objective is achieved if* $\mathcal{G}^f(\sigma(t)) = (\mathcal{N}^f, \mathcal{E}^f(\sigma(t)))$ *with* $\sigma : [0, \infty) \to \mathcal{P}$ *is UQSC and has an edge dwell-time* $\tau_D^f > 0$*.*

5 Conclusions

This paper has presented a new distributed nonlinear control design for strong output agreement of multi-agent systems modelled by double-integrators under dynamically changing topologies.. With the new design, the information exchange topology of the large-scale interconnected multi-agent system is allowed to be directed and switching, as long as a mild connectivity condition is satisfied. By appropriately designing the distributed control law, the velocities of the agents can be restricted to be within any neighborhood of the origin, which is of practical interest. As an application, a distributed formation control algorithm has also been developed for groups of unicycle mobile robots with time-variable topologies and relative position measurements. The singularity problem caused by the nonholonomic constraint is solved by properly designing the distributed control law.

This paper has studied the distributed nonlinear control of multi-agent systems under constraints. For further practical implementation of the result, several related problems are of special interest. These problems may be caused by the ubiquitous communication constraints such as time-delay, quantization and sensor noise in multi-agent systems working in complex environments. In practice, it is possible that two agents disagree on the desired relative distance value, e.g., $d_{xij} \neq d_{xji}$. In this case, an external disturbance could be introduced to the system, and the practical stability problem is meaningful. Recently, the notion of input-to-state stability (ISS) and the nonlinear small-gain theorem have been applied to the control of multi-agent systems [35]. Considering their power in handling disturbances, ISS small-gain methods could provide potential solutions to distributed nonlinear control under external disturbances. The distributed control law developed in this paper uses absolute velocity instead of relative velocity information of the agents, which might be restrictive for practical applications. By considering the velocities as internal states, the recently developed distributed output agreement result for nonlinear systems [36], which also uses the nonlinear small-gain methods, could be helpful in solving the problem. Moreover, the distributed control problem for more general nonlinear systems in the coexistence of communication constraints, dynamic uncertainties and complex physical interconnections are expected to be solved by refining the existing nonlinear small-gain results; see, e.g., [25] for the recent development of the literature. These problems of both theoretical and practical importance are under current investigations.

A A Technical Lemma

Lemma 2 *Consider the following first-order system*

$$\dot{\varsigma} = \alpha(\varsigma) \tag{66}$$

where $\varsigma \in \mathbb{R}$ *is the state and* α *is non-increasing and locally Lipschitz and satisfies* $\alpha(0) = 0$, $r\alpha(r) < 0$ *for all* $r \neq 0$. *There exists* $\beta \in \mathcal{IL}$ *such that for any* $\varsigma_0 \in \mathbb{R}$, *with initial condition* $\varsigma(0) = \varsigma_0$, *it holds that*

$$\varsigma(t) \leq \beta(\varsigma_0, t) \tag{67}$$

for all $t \geq 0$.

Proof. Denote $\varsigma^*(\varsigma_0, t)$ as the solution of system (66) with initial condition $\varsigma(0) = \varsigma_0$. Define $\beta'(s,t) = \varsigma^*(s,t)$ and $\beta''(s,t) = -\varsigma^*(-s,t)$ for $s, t \in \mathbb{R}_+$.

Consider the case of $\varsigma(0) \geq 0$.

Because α is locally Lipschitz, for any specified $\overline{\varsigma} > 0$, there exists a constant $k_\alpha > 0$ such that $\alpha(s) \geq -k_\alpha s$ for $s \leq \overline{\varsigma}$ and thus

$$\dot{\varsigma}(t) \geq -k_\alpha \varsigma(t)$$

for $0 \leq \varsigma(t) \leq \overline{\varsigma}$.

For any specified $T > 0$, define

$$\alpha_1(s) = e^{-k_\alpha T} \min\{s, \overline{\varsigma}\}$$

for $s \in \mathbb{R}_+$. Then, α_1 is continuous and positive definite.

If $\varsigma(0) \leq \overline{\varsigma}$, then

$$\varsigma(T) \geq \varsigma(0) e^{-k_\alpha T} \geq \alpha_1(\varsigma(0)).$$

Consider the case of $\varsigma(0) > \overline{\varsigma}$. If there is a time $0 < t' \leq T$ such that $\varsigma(t') = \overline{\varsigma}$, then

$$\varsigma(T) \geq \varsigma(t') e^{-k_\alpha (T - t')} \geq \overline{\varsigma} e^{-k_\alpha T} > \alpha_1(\varsigma(0));$$

otherwise,

$$\varsigma(T) > \overline{\varsigma} > \overline{\varsigma} e^{-k_\alpha T} > \alpha_1(\varsigma(0)).$$

According to the definition of β', for the specified $T > 0$, it holds that $\beta'(s,T) \geq \alpha_1(s)$. Because of the nonincreasing property of $\varsigma(t)$ with $\varsigma(0) \geq 0$, we have

$$\beta'(s,t) \geq \alpha_1(s) \tag{68}$$

for $t \in [0,T]$.

For any specified $T > 0$, define

$$\alpha_2(s) = \max\left\{\frac{1}{2}s, s + T \max_{\frac{1}{2}s \leq \tau \leq s} \alpha(\tau)\right\}$$

for $s \in \mathbb{R}_+$. It can be verified that α_2 is continuous, positive definite and less than Id.

If $\varsigma(T) \geq \frac{1}{2}\varsigma(0)$, then $\frac{1}{2}\varsigma(0) \leq \varsigma(t) \leq \varsigma(0)$ for $0 \leq t \leq T$, and

$$\dot{\varsigma}(t) \leq \max_{\frac{1}{2}\varsigma(0) \leq \tau \leq \varsigma(0)} \alpha(\tau).$$

Then, we have

$$\begin{aligned}
\varsigma(T) &\leq \varsigma(0) + \int_0^T \max_{\frac{1}{2}\varsigma(0) \leq \tau \leq \varsigma(0)} \alpha(\tau) dt \\
&= \varsigma(0) + T \max_{\frac{1}{2}\varsigma(0) \leq \tau \leq \varsigma(0)} \alpha(\tau) \\
&\leq \alpha_2(\varsigma(0)).
\end{aligned}$$

If $\varsigma(T) < \frac{1}{2}\varsigma(0)$, then $\varsigma(T) < \alpha_2(\varsigma(0))$ automatically. According to the definition of β', for the specified $T > 0$, it holds that $\beta'(s,T) \leq \alpha_2(s)$. Because of the nonincreasing property of $\varsigma(t)$ with $\varsigma(0) \geq 0$, we have

$$\beta'(s,t) \leq \alpha_2(s) \tag{69}$$

for $t \in [T, \infty)$.

It can be directly verified that $\beta' \in \mathcal{KL}$ and $\beta'(s,0) = s$ for $s \geq 0$. By also using (68) and (69), we can prove $\beta' \in \mathcal{I}^+\mathcal{L}$. Due to symmetry, we can also prove $\beta'' \in \mathcal{I}^+\mathcal{L}$. Thus, $\beta \in \mathcal{IL}$.

Define $\beta(r,t) = \beta'(r,t)$ for $r \geq 0$, $t \geq 0$ and $\beta(r,t) = -\beta''(-r,t)$ for $r \leq 0$, $t \geq 0$. Then, (67) holds and $\beta \in \mathcal{IL}$. This ends the proof. ♢

B Proof of Proposition 1

B.1 Property 1

Denote $S^a(\overline{v}) = \{(\eta, \zeta) : \zeta \leq \overline{\psi}(\eta - \overline{v})\}$ and $S^b(\underline{v}) = \{(\eta, \zeta) : \zeta \geq \underline{\psi}(\eta - \overline{v})\}$. Then, $S(\underline{v}, \overline{v}) = S^a(\overline{v}) \cap S^b(\underline{v})$. If both $S^a(\overline{v})$ and $S^b(\underline{v})$ are invariant sets of system (13)–(14) and $\underline{\psi}(r) \leq \overline{\psi}(r)$ for all $r \in \mathbb{R}$, then $S(\underline{v}, \overline{v})$ is an invariant set. In the following procedure, we find appropriate $\overline{\psi}$ such that $S^a(\overline{\upsilon})$ is an invariant set. Function $\underline{\psi}$ can be found in the same way.

For a nonincreasing and locally Lipschitz function ϕ satisfying (15) and (16), there exists a function $\overline{\phi} : \mathbb{R} \to \mathbb{R}$ which is strictly decreasing and continuously differentiable on $(-\infty, 0) \cup (0, \infty)$ such that $\overline{\phi}(0) = 0$, $\overline{\phi}(r) \geq \phi(r)$ for $r \in \mathbb{R}$ and

$$\inf_{r\in\mathbb{R}}\{\min \partial\overline{\phi}(r)\} \geq \inf_{r\in\mathbb{R}}\{\min \partial\phi(r)\} - \epsilon \tag{70}$$

for any specified arbitrarily small $\epsilon > 0$.

Define

$$\overline{\psi}(r) = \max\left\{c\overline{\phi}(r) : c \in [c_2, c_1]\right\} \tag{71}$$

where c_1 and c_2 are constants satisfying $0 < c_2 < 1 < c_1$ to be determined later. Then, $\overline{\psi}$ is strictly decreasing and continuously differentiable on $(-\infty, 0) \cup (0, \infty)$ and $\overline{\psi}(0) = 0$.

Define $\tilde{\zeta} = \zeta - \overline{\psi}(\eta - \overline{\upsilon})$. When $\zeta \geq \overline{\psi}(\eta - \overline{\upsilon})$, directly taking the derivative of $\tilde{\zeta}$ yields:

$$\begin{aligned}
\dot{\tilde{\zeta}} &\in \left\{\dot{\zeta} - \overline{\psi}^d \dot{\eta} : \overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon})\right\} \\
&= \left\{\varphi(\zeta - \phi(\eta - \upsilon)) - \overline{\psi}^d \zeta : \overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon})\right\} \\
&\subseteq \Big\{\varphi(\zeta - \phi(\eta - \upsilon)) - \overline{\psi}^d \zeta : \\
&\qquad\qquad \overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon}), \underline{\upsilon} \leq \upsilon \leq \overline{\upsilon}\Big\} \\
&= \Bigg\{-(k_\varphi + \overline{\psi}^d)\left(\zeta - \frac{k_\varphi \phi(\eta - \upsilon)}{k_\varphi + \overline{\psi}^d}\right) + \tilde{\varphi}(\zeta - \phi(\eta - \upsilon)) : \\
&\qquad\qquad \overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon}), \underline{\upsilon} \leq \upsilon \leq \overline{\upsilon}\Bigg\} \\
&:= F_{\tilde{\zeta}}(\eta, \zeta, \underline{\upsilon}, \overline{\upsilon})
\end{aligned} \tag{72}$$

where $k_\varphi := -\sup\{\partial\varphi(r) : r \in \mathbb{R}\}$ and $\tilde{\varphi}(r) := \varphi(r) + k_\varphi r$ for $r \in \mathbb{R}$. Clearly,

$$\tilde{\varphi}(r) r \leq 0 \tag{73}$$

for $r \in \mathbb{R}$.

Denote $k_{\overline{\phi}} = -\inf_{r\in\mathbb{R}}\{\min \partial\overline{\phi}(r)\}$. With condition (17) and (70) satisfied, we can choose $\overline{\phi}$ such that $0 < 4k_{\overline{\phi}} \leq k_\varphi$. Choose $c_1 = k_\varphi / 2k_{\overline{\phi}}$. Then, $k_{\overline{\phi}} c_1^2 - k_\varphi c_1 + k_\varphi \leq 0$, i.e., $k_\varphi/(k_\varphi - c_1 k_{\overline{\phi}}) \leq c_1$. Clearly, $c_1 \geq 2$. Choose $c_2 \leq 1$.

The definition of $\overline{\psi}$ in (71) implies

$$\partial\overline{\psi}(r) \subseteq \left\{c\overline{\phi}^d : c \in [c_2, c_1], \overline{\phi}^d \in \partial\overline{\phi}(r)\right\},$$

and thus $\inf_{r\in\mathbb{R}}\{\min \partial\overline{\psi}(r)\} \geq -c_1 k_{\overline{\phi}}$. By also using $\sup_{r\in\mathbb{R}}\{\max \partial\overline{\psi}(r)\} \leq 0$ (due to the strictly decreasing of $\overline{\psi}$), we can prove

$$c_2 \leq \frac{k_\varphi}{k_\varphi + \overline{\psi}^d} \leq c_1 \tag{74}$$

for $\overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon})$.

Using $\upsilon \leq \overline{\upsilon}$ and the nonincreasing of ϕ and $\overline{\phi}$, from (74), we have

$$\frac{k_\varphi \phi(\eta - \upsilon)}{k_\varphi + \overline{\psi}^d} \leq \max\left\{c\overline{\phi}(\eta - \overline{\upsilon}) : c \in [c_2, c_1]\right\} = \overline{\psi}(\eta - \overline{\upsilon}),$$

which implies

$$\zeta - \frac{k_\varphi \phi(\eta - \upsilon)}{k_\varphi + \overline{\psi}^d} \geq \tilde{\zeta} \tag{75}$$

for $\overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon})$.

Based on the definitions of $\overline{\psi}^d$ and c_1, we also have

$$k_\varphi + \overline{\psi}^d \geq k_\varphi + \inf_{r\in\mathbb{R}}\{\min \partial\overline{\psi}(r)\} = k_\varphi - c_1 k_{\overline{\phi}} = \frac{1}{2}k_\varphi \tag{76}$$

for $\overline{\psi}^d \in \partial\overline{\psi}(\eta - \overline{\upsilon})$.

When $\zeta \geq \overline{\psi}(\eta - \overline{\upsilon})$, $\zeta \geq \overline{\psi}(\eta - \upsilon)$ for all $\underline{\upsilon} \leq \upsilon \leq \overline{\upsilon}$, and thus $\tilde{\varphi}(\zeta - \phi(\eta - \upsilon)) \leq 0$ due to property (73). Then, based on (72), (75) and (76), it can be proved that

$$\max_{f_{\tilde{\zeta}} \in F_{\tilde{\zeta}}(\eta, \zeta, \underline{\upsilon}, \overline{\upsilon})} f_{\tilde{\zeta}} \leq -\frac{1}{2}k_\varphi \tilde{\zeta} \tag{77}$$

when $\zeta \geq \overline{\psi}(\eta - \overline{\upsilon})$, i.e., $\tilde{\zeta} \geq 0$. This guarantees the invariance of set $S^a(\overline{\upsilon})$.

Following a similar approach, we can also find $\underline{\psi} : \mathbb{R} \to \mathbb{R}$ such that it is strictly decreasing and continuously differentiable on $(-\infty, 0) \cup (0, \infty)$ and satisfies $\underline{\psi}(0) = 0$ and $\underline{\psi} \leq \overline{\psi}(r)$ for all $r \in \mathbb{R}$, and prove that $S^b(\underline{\upsilon})$ is an invariant set.

B.2 Property 2

We present only the proof of the second inequality in (18). The first inequality in (18) can be proved in the same way.

We first consider the case in which $\phi(r) \to \infty$ as $r \to -\infty$. In this case, $\overline{\psi}(r) \to \infty$ as $r \to -\infty$ according to the definition of $\overline{\psi}$ in (71). In this case, for any $(\eta(0), \zeta(0))$, one can always find a $\overline{\mu}$ such that the second inequality in (18) holds with $t_1 = 0$.

If the condition for the first case is not satisfied, then there exist constants $\phi^u > 0$ and $2/3 < \phi^\delta < 1$ such that $\phi(r) \leq \phi^u$ for all $r \in \mathbb{R}$ and one can find an r^* satisfying $\phi(r^*) \geq \phi^\delta \phi^u$. According to the definition of $\overline{\psi}$ in (71), it holds that $\overline{\psi}(r^*) \geq c_1 \phi^\delta \phi^u$, where $c_1 \geq 2$, and thus $\overline{\psi}(r^*) \geq 4\phi^u/3$.

Define $\tilde{\zeta} = \zeta - \phi^u$. When $\zeta \geq \phi^u$, taking the derivative of $\tilde{\zeta}$ yields

$$\dot{\tilde{\zeta}} = \dot{\zeta} = \varphi(\zeta - \phi(\eta - \upsilon)) \leq \phi(\zeta - \phi^u) = \varphi(\tilde{\zeta})$$

where φ satisfies (15), (16) and (17). Then, there exists a $\beta \in \mathcal{KL}$ such that for any $\zeta(0) \geq \phi^u$,

$$\tilde{\zeta}(t) \leq \beta(\tilde{\zeta}(0), t)$$

for all $t \geq 0$. According to [51, Lemma 8], there exist $\alpha_{\beta 1}, \alpha_{\beta 2} \in \mathcal{K}_\infty$ such that $\beta(s,t) \leq \alpha_{\beta 1}(s)\alpha_{\beta 2}(e^{-t})$ for all $s, t \in \mathbb{R}_+$ and thus there exists a finite time t_1 such that $\beta(\tilde{\zeta}(0), t_1) \leq \phi^u/3$, which guarantees $\tilde{\zeta}(t_1) \leq \phi^u/3$, i.e., $\zeta(t_1) \leq 4\phi^u/3$. During finite time interval $[0, t_1]$, the boundedness of $\zeta(t)$ implies the boundedness of $\eta(t)$. Then, one can find a $\overline{\mu}$ such that the second inequality in (18) holds.

B.3 Property 3

For $t \in [0, T]$, it holds that

$$\dot{\eta}(t) = \zeta(t) \leq \overline{\psi}(\eta(t) - \overline{\sigma}).$$

Define $\varsigma(t)$ as the solution of the initial value problem

$$\dot{\varsigma}(t) = \overline{\psi}(\varsigma(t) - \overline{\sigma}) \tag{78}$$

with $\varsigma(0) = \eta(0)$. By using the comparison principle (see e.g. [26]), it can be proved that

$$\eta(t) \leq \varsigma(t) \tag{79}$$

for $t \in [0, T]$.

Note that $\overline{\psi}$ is locally Lipschitz and satisfies $r\overline{\psi}(r) < 0$ for $r \neq 0$. Define $\tilde{\varsigma} = \varsigma - \overline{\sigma}$. Then, (78) implies $\dot{\tilde{\varsigma}}(t) = \overline{\psi}(\tilde{\varsigma}(t))$. By using Lemma 2 in the Appendix, there exists a $\overline{\beta}_1 \in \mathcal{IL}$ such that $\tilde{\varsigma}(t) \leq \overline{\beta}_1(\tilde{\varsigma}(0), t)$ for $t \in [0, T]$, i.e.,

$$\varsigma(t) \leq \overline{\beta}_1(\varsigma(0) - \overline{\sigma}, t) + \overline{\sigma}.$$

The second inequality in (19) is proved by using $\varsigma(0) = \eta(0)$ and $\eta(t) \leq \varsigma(t)$ for $t \in [0, T]$. The first inequality in (19) can be proved in the same way.

B.4 Property 4

Define $\underline{\upsilon}^* = \min\{\underline{\mu}_0, \eta(0), \underline{\upsilon}\}$ and $\overline{\upsilon}^* = \max\{\overline{\mu}_0, \eta(0), \overline{\upsilon}\}$. Then, $(\eta(0), \zeta(0)) \in S(\underline{\upsilon}^*, \overline{\upsilon}^*) \cap \{(\eta, \zeta) : \underline{\upsilon}^* \leq \eta \leq \overline{\upsilon}^*\} := \breve{S}(\underline{\upsilon}^*, \overline{\upsilon}^*)$.

Property 1 of Proposition 1 shows that $S(\underline{v}, \overline{v})$ is an invariant set of system (13)–(14) as long as $v \in [\underline{v}, \overline{v}]$. Note that $v \in [\underline{v}, \overline{v}] \subseteq [\underline{v}^*, \overline{v}^*]$. Thus, $S(\underline{v}^*, \overline{v}^*)$ is an invariant set of system (13)–(14) with $v \in [\underline{v}^*, \overline{v}^*]$, which is guaranteed by $v \in [\underline{v}, \overline{v}]$. The invariance of $\breve{S}(\underline{v}^*, \overline{v}^*)$ can be proved by using property 3 given by Proposition 1.

In the following proof, we adopt some idea from kinematics of the plane translational motion of a rigid body; see e.g., [17]. The basic idea is shown in Figure 2. Define

$$
\begin{aligned}
&\eta^d = \zeta, \\
&\zeta^d = \varphi(\zeta - \phi(\eta - v)), \\
&v = [\eta^d, \zeta^d]^T, \\
&v_1 = \left[\min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\zeta^d}{\overline{\psi}^d}, \zeta^d \right]^T, \\
&v_2 = \left[\eta^d - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\zeta^d}{\overline{\psi}^d}, 0 \right]^T.
\end{aligned}
$$

Clearly, $v_2(t) = v(t) - v_1(t)$.

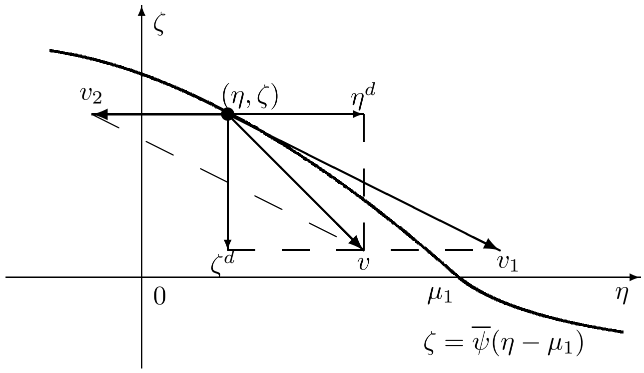

Figure 2: The motion of the point (η, ζ) and the rigid body $\zeta = \overline{\psi}(\eta - \mu)$: v is the velocity of the point which is the composition of η^d and ζ^d and also the composition of v_1 and v_2; v_1 represents the relative velocity of the point along the rigid body and v_2 represents the translational motion velocity of the rigid body.

For any specified $\overline{\mu}$, if $v \in [\underline{v}^*, \overline{\mu}]$, then $S(\underline{v}^*, \overline{\mu})$ is an invariant set. From (77), for any (η, ζ) satisfying $\zeta = \overline{\psi}(\eta - \mu)$, it holds that

$$
\eta^d - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\zeta^d}{\overline{\psi}^d} \leq 0, \tag{80}
$$

i.e.,

$$\zeta - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \upsilon))}{\overline{\psi}^d} \leq 0 \tag{81}$$

for all $\upsilon \in [\underline{\upsilon}^*, \overline{\mu}]$. Thus, it can be concluded that

$$\zeta - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d} \leq 0 \tag{82}$$

for any (η, ζ) satisfying $\zeta = \overline{\psi}(\eta - \overline{\mu})$.

Then, for any (η, ζ) satisfying $\zeta = \overline{\psi}(\eta - \overline{\mu})$, it holds that

$$\begin{aligned}
&\eta^d - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\zeta^d}{\overline{\psi}^d} \\
=&\zeta - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\upsilon}))}{\overline{\psi}^d} \\
=&\zeta - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d} \\
&- \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\upsilon})) - \varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d} \\
\leq& - \min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\upsilon})) - \varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d},
\end{aligned}$$

where we used (82) for the inequality.

In the case of $\overline{\mu} \geq \overline{\upsilon}$, by using the continuous and strictly decreasing properties of φ and ϕ, one can always find a positive definite, non-decreasing, locally Lipschitz $\overline{\alpha}_2^a$ such that

$$\min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\upsilon})) - \varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d} \geq \overline{\alpha}_2^a(\overline{\mu} - \overline{\upsilon})$$

for $(\eta, \zeta) \in \breve{S}(\underline{\upsilon}^*, \overline{\upsilon}^*)$ and $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}]$.

In the case of $\overline{\mu} < \overline{\upsilon}$, because φ and ϕ are locally Lipschitz and strictly decreasing, one can find a positive definite, non-decreasing, locally Lipschitz $\overline{\alpha}_2^b$ such that

$$\min_{\overline{\psi}^d \in \partial\overline{\psi}(\eta)} \frac{\varphi(\zeta - \phi(\eta - \overline{\upsilon})) - \varphi(\zeta - \phi(\eta - \overline{\mu}))}{\overline{\psi}^d} \geq -\overline{\alpha}_2^b(\overline{\upsilon} - \overline{\mu})$$

for $(\eta, \zeta) \in \breve{S}(\underline{\upsilon}^*, \overline{\upsilon}^*)$ and $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}]$.

Define

$$\overline{\alpha}_2(r) = \begin{cases} -\overline{\alpha}_2^a(r) & \text{for } r \geq 0; \\ \overline{\alpha}_2^b(-r) & \text{for } r < 0. \end{cases}$$

Then, $\overline{\alpha}_2(0) = 0$, $r\overline{\alpha}_2(r) < 0$ for all $r \neq 0$, and $\overline{\alpha}_2$ is non-increasing and locally Lipschitz.

Define $\overline{\varsigma}(t)$ as the solution of the initial value problem

$$\dot{\overline{\varsigma}}(t) = \overline{\alpha}_2(\overline{\varsigma}(t) - \overline{\upsilon}) \tag{83}$$

with initial condition $\overline{\varsigma}(0) = \overline{\mu}_0$. Then, $\zeta(t) \leq \overline{\psi}(\eta(t) - \overline{\varsigma}(t))$ for $t \geq 0$. If $\overline{\mu}(t) \geq \overline{\sigma}(t)$ for $t \geq 0$, then $\zeta(t) \leq \overline{\psi}(\eta(t) - \overline{\mu}(t))$ for $t \geq 0$.

Similarly, one can find a non-increasing, locally Lipschitz $\underline{\alpha}_2$ which satisfies $\underline{\alpha}_2(0) = 0$, $r\underline{\alpha}_2(r) < 0$ for all $r \neq 0$, such that $\zeta(t) \geq \underline{\psi}(\eta(t) - \underline{\mu}(t))$ for $t \geq 0$, if $\underline{\mu}(t) \leq \underline{\sigma}(t)$ for $t \geq 0$, where $\underline{\varsigma}(t)$ is the solution of the initial value problem

$$\dot{\underline{\varsigma}}(t) = \underline{\alpha}_2(\underline{\varsigma}(t) - \underline{\upsilon}) \tag{84}$$

with initial condition $\underline{\varsigma}(0) = \underline{\mu}_0$.

Define $\overline{\alpha}'_2 = \max\{\overline{\alpha}_2(r), \underline{\alpha}_2(r)\}$ and $\underline{\alpha}'_2 = \min\{\overline{\alpha}_2(r), \underline{\alpha}_2(r)\}$ for $r \in \mathbb{R}$. Define $\overline{\mu}(t)$ and $\underline{\mu}(t)$ as the solutions of the initial value problems

$$\dot{\overline{\mu}}(t) = \overline{\alpha}'_2(\overline{\mu}(t) - \overline{\upsilon})$$
$$\dot{\underline{\mu}}(t) = \underline{\alpha}'_2(\underline{\mu}(t) - \underline{\upsilon})$$

with initial conditions $\overline{\mu}(0) = \overline{\mu}_0$ and $\underline{\mu}(0) = \underline{\mu}_0$. Then, the comparison principle can guarantee $\overline{\mu}(t) \leq \overline{\sigma}(t)$ and $\underline{\mu}(t) \leq \underline{\sigma}(t)$. Moreover, $\overline{\mu}(t) \geq \underline{\mu}(t)$ for $t \geq 0$.

By using Lemma 2 in the Appendix, one can find $\underline{\beta}_2, \overline{\beta}_2 \in \mathcal{IL}$ such that (20) holds.

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